Construction

Construct a 2^3 design in A, B and C and then

$$D = AB \quad F = BC$$
$$E = AC \quad G = ABC$$

} Generators

$I = ABD = ACE = BCF = ABCG$

$I^2 = BCDE = ACDF = CDG = ABEF = BEG = AFG$

$I^3 = DEF = ADEG = BDFG = CEFG$

$I^4 = ABCDEFG$

Neglecting interactions of order three and higher.

$\hat{l}_A \rightarrow A + BD + CE + FG$

$\hat{l}_B \rightarrow B + AD + CF + EG$

$\hat{l}_C \rightarrow C + AE + BF + DG$

$\hat{l}_D \rightarrow D + AB + CG + EF$

$\hat{l}_E \rightarrow E + AC + BG + DF$

$\hat{l}_F \rightarrow F + BC + AG + DE$

$\hat{l}_G \rightarrow G + CD + BE + AF$

There are 16 possible 2^{7-4} fractions according to the generator

$$D = \pm AB \quad F = \pm BC$$
$$E = \pm AC \quad G = \pm ABC$$

For estimating main effects free of aliasing with two-factor interactions, run a new 2^{7-4} fraction where $D = -AB$, $F = -BC$, $E = -AC$, $G = ABC$ } Generators

Hence; $I = -ABD = -ACE = -BCF = ABCG$

$I^2 = BCDE = ACDF = -CDG = ABEF = -BEG = -AFG$

$I^3 = -DEF = ADEG = BDFG = CEFG$

$I^4 = -ABCDEFG$

Hence.

$$\mathring{l}_A \to A - BD - CE - FG$$

$$\mathring{l}_B \to B - AD - CF - EG$$

$$\vdots$$

$$\hat{l}_G \to G - CD - BE - AF$$

$$\frac{\hat{l}_A + \mathring{l}_A}{2} \to A$$

$$\vdots$$

$$\frac{\hat{l}_G + \mathring{l}_G}{2} \to G$$

Estimating one main effect and all interactions involved with the corresponding factor. Assume factor D is the interesting factor. Run a new 2^{7-4} fractions where $D = -AB$, $E = AC$, $F = BC$ and $G = ABC$.

We get. $I = -ABD = ACE = BCF = ABCG$

$I^2 = -BCDE = -ACDF = -CDG = ABEF = BEG = AFG$

$I^3 = -DEF = -ADEG = -BDFG = CEFG$

$I^4 = -ABCDEFG$

$$\tilde{l}_A \Rightarrow A - BD + CE + FG$$

$$\tilde{l}_B \Rightarrow B - AD - CF + EG$$

$$\vdots$$

$$\tilde{l}_D \to D - AB - CG - EF$$

Hence $\frac{\hat{l}_D + \tilde{l}_D}{2} \to D$

$$\frac{\hat{l}_A - \tilde{l}_A}{2} \to BD$$

$$\hat{D} = \frac{22.5 + 25.25}{2} = 23.88$$

$$\hat{BD} = \frac{3.5 - 0.75}{2} = 1.37$$

Fold-over of resolution III designs.

Start with 2^{7-4}_{III} where $D = AB$, $E = AC$, $F = BC$; $G = ABC$

Let $H = I_8$ then.

$I_8 = ABD = ACE = BCF = ABCG = H$

Add eight more runs where $D = -AB$, $E = -AC$, $F = -BC$ and $G = ABC$ and $I_8 = -H$. This is the same as switching all signs in the columns.

For the eight last runs we have

$I_8 = -ABD = -ACE = -BCF = ABCG = -H$

For the 16 run design: $I_{16} = ABCG = ABDH = ACEH = BCFH$ a resolution IV design. Verify I^2, I^3 and I^4.

Blocking in fractional factorial designs.

Example: 2^{5-1} in two blocks after AB

$I = ABCDE$. Then $I\,AB = CDE$ is also confounded with the block effect. Example. 2^{5-1} in four blocks using AC and BC

Then AB is confounded with block and also

$I\,AC = BDE$, $I\,BC = ADE$, $I\,AB = CDE$.